

Research paper

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## Michell structures formed on surfaces of revolution

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**Abstract** The paper formulates the Michell-like problems for surface gridworks. Particular attention is devoted to the problem of designing the lightest fully stressed gridworks formed on surfaces of revolution. In the examples considered, the gridworks are subjected to torsion. Proof is given that the circular meridian is a minimizer of the weight (or volume) functional of a shell subjected to torsion, thus justifying the original Michell conjecture according to which just the spherical twisting shell is the lightest. The proof is based on the methods of the classical variational calculus and thus can be viewed as elementary. This result is confirmed by a direct comparison of the exact formulae for the weight of a spherical Michell shell with the exact formulae for the weights of optimal conical and cylindrical shells with the same fixed boundaries.

**Keywords** Michell structures, Michell's problem

The specification is done in two steps. In the first step, one should allow the models of the bodies under investigation to consist of shells. Next, one should assume that the volume occupied by the structure (not the only shell) material is constant. This passage makes the minimized functional independent of the choice of shell. The problem becomes similar to an equilibrium problem of a twisting material, as pointed out by Hest and Koenig (see literature [1978, Sect. 2.3, Pagan 1972, Tolson and Boushka 1968, and Lewiński and Trigo 2001]). The stress distribution can be viewed as a stream function.

The following question arises: Can one apply a similar approach to formulate the Michell surface problem by appropriate specification of the minimum compliance problem of inextensible shells? The affirmative answer to the problem is given by Lewiński and Trigo (2004). However, one should remember an essential difference between the equilibrium problem of a plane body and

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