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Abstract

By endowing El Fatmi's theories of bars with first-order warping functions due to torsion and shear, a family of theories of bars, of various applicability ranges, is effectively constructed. The theories thus formed concern bars of arbitrary cross-sections; they are reformulations of the mentioned theories by El Fatmi and theories by Kim and Kim, Librescu and Song, Vlasov and Timoshenko. The Vlasov-like theory thus developed is capable of describing the torsional buckling and lateral buckling phenomena of bars of both solid and thin-walled cross-sections, which reflects the non-trivial correspondence, noted by Wagner and Gruttmann, between the torsional St.Venant's warping function and the contour-wise defined warping functions proposed by Vlasov. Moreover, the present paper delivers an explicit construction of the constitutive equations of Timoshenko's theory; the equations linking transverse forces with the measures of transverse shear turn out to be coupled for all bars of asymmetric cross-sections. The modeling is hierarchical: the warping functions are numerically constructed by solving the three underlying 2D scalar elliptic problems, providing the effective characteristics for the 1D models of bars. The 2D and 1D problems are indissolubly bonded, thus forming a unified scientific tool, deeply rooted in the hitherto existing knowledge on elasticity of elastic straight bars.