Three-phase plane composites of minimal elastic stress energy: High-porosity structures.

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The paper establishes exact lower bound on the effective elastic energy of two-dimensional, three-material composite subjected to the homogeneous, anisotropic stress. It is assumed that the materials are mixed with given volume fractions and that one of the phases is degenerated to void, i.e., the effective composite is porous. Explicit formula for the energy bound is obtained using the translation method enhanced with additional inequality expressing certain property of stresses. Sufficient optimality conditions of the energy bound are used to set the requirements which have to be met by the stress fields in each phase of optimal effective material regardless of the complexity of its microstructural geometry. We show that these requirements are fulfilled in a special class of microgeometries, so-called laminates of a rank. Their optimality is elaborated in detail for structures with significant amount of void, also referred to as high-porosity structures. It is shown that geometrical parameters of optimal multi-rank, high-porosity laminates are different in various ranges of volume fractions and anisotropy level of external stress. Non-laminate, three-phase microstructures introduced by other authors and their optimality in high-porosity regions is also discussed by means of the sufficient conditions technique. Conjectures regarding low-porosity regions are presented, but full treatment of this issue is postponed to a separate publication. The corresponding "G-closure problem" of a three-phase isotropic composite is also addressed and exact bounds on effective isotropic properties are explicitly determined in these regions where the stress energy bound is optimal.