

Research paper

Struct Multidisc Optim 28, 20–30 (2004)
DOI 10.1007/s00158-004-0413-7

Michell structures formed on surfaces of revolution

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Abstract The paper formulates the Michell-like problems for surface gridworks. Particular attention is devoted to the problem of designing the lightest fully stressed problems formed on surfaces of revolution. In the examples considered, the gridworks are subjected to torsion. Proof is given that the circular meridian is a minimizer of the weight (or volume) functional of a shell subjected to torsion, thus justifying the original Michell conjecture according to which just the spherical twisting shell is the lightest. The proof is based on the methods of the classical variational calculus and thus can be viewed as elementary. This result is confirmed by a direct comparison of the exact formulae for the weight of a spherical Michell shell with the exact formulae for the weights of optimal conical and cylindrical shells with the same fixed boundaries.

Keywords Michell structures, Michell's conjecture

The specification is done in two steps. In the first step, one should allow the model of the member material to assume existing values, thus disregarding the weaker material into voids. Next, one should assume that the volume occupied by the member (not the only shell) material is constant. This passage makes the minimized functional independent of the choice of model. The problem becomes similar to an equilibrium problem of a twisting material, as pointed out by Biot and Dornay (see Dornay 1978, Sect. 2.3; Pagan 1972; Tolson and Isakovic 1968, and Lewiński and Trzga 2001). The stress should not be viewed as a stress.

The following question arises: Can one apply a similar approach to formulate the Michell surface problems – by appropriate specification of the unknown complex problem of inextensible shells? The affirmative answer to the problem is given by Lewiński and Trzga (2004). However, one should remember an essential difference between the equilibrium problem of a plane body and

Michell structures formed on surfaces of revolution, T. Lewiński, *Structural and Multidisciplinary Optimization* 28, 20–30 (2004)

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