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RESEARCH PAPER

## On the solution of the three forces problem and its application in optimal designing of a class of symmetric plane frameworks of least weight

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**Abstract** Two problems of minimum weight design of plane trusses are dealt with. The first problem concerns construction of the lightest fully stressed truss subject to three self-equilibrated forces applied at three given points. This problem has been solved analytically by H.S.Y. Chan in 1966. This analytical solution is re-derived in the present paper. It compares favourably with new numerical solutions found here by the method developed recently by the first author. The solution to the three forces problem paves the way to half-analytical as well as numerical solutions to the problem of minimum weight design of plane symmetric frameworks transmitting two symmetrically located vertical forces across fixed supports lying along the line linking the points of application of the forces.

Two-plane forces applied at three given points. This problem will not be solved completely, it involves many parameters. Nevertheless, an important class of its solutions, belonging to the Michell (1904) class, will be considered. Solution of this problem will make it possible to solve a minimum weight design problem of fully stressed symmetric frameworks transmitting two vertical forces to two fixed hinge supports lying on the line of application of the forces.

The three-forces problem was discussed by Chan (1966), this paper being unavailable for the present authors, and these results are cited in Flügge (1975) and in the reports by Chan (1963), 1966, 1968 being available for the present authors. Some suggestions of how to solve the three-forces problem can be found in Dornbusch (1981) and Mikhlin.

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